

The program is composed on the basis of the sample program in Mathematics for secondary education institutions approved by the Ministry of Education and Science of Ukraine (№1/11-6611 dd. 23.12.2004).

The program was developed by Tatyana Spychak, Candidate of Pedagogical Sciences, Associate Professor of the Department of Natural Science, and approved by minutes № 6 at the meeting of the Department of Natural Science on February 26, 2021.

Head of exam Committee



Tatyana SPYCHAK

GENERAL INFORMATION

The program of entrance examinations in Mathematics for applicants with a basic secondary education is developed in accordance with the Laws of Ukraine “On Education”, “On Higher Education”, the Conditions of Admission to Higher Educational Institutions of Ukraine. The program is composed on the basis of the sample program in Mathematics for secondary education institutions approved by the Ministry of Education and Science of Ukraine (№1/11-6611 dd. 23.12.2004) and includes all sections of school curricula.

An entrance examination and interview at Kherson State Maritime Academy in Mathematics is held for admission of the applicants who are entitled to participate in competitive selection in accordance with the results of entrance examinations. Category of the persons is defined in the Regulations of Admission to KSMA.

The object of an entrance examination is the estimation of the level of applicants' knowledge in Mathematics to participate in competitive selection.

The program consists of 6 sections. The 1st section contains a list of basic mathematical notions and facts to be known by an applicant (to be able to use them during solving mathematical problems). The 2nd section is composed of theorems and formulas which are the basis of the examination theoretical part. Mathematical skills to be known by an applicant are enumerated in the 3rd section. In order to prepare better for an interview, a list of typical mathematical problems for applicants' self-preparation are provided in the 4th section. The 5th section contains criteria of applicants' knowledge and skills assessment. Proposed list of reference for preparation for an examination and interview is provided in the 6th section. Moreover, while preparing for entrance examinations, all textbooks and manuals in Algebra and Geometry recommended by the current program and developed for regular schools in Mathematics can be used.

The aim of entrance examination and interview in Mathematics is to estimate applicants' knowledge and skills. An applicant should demonstrate:

- precise knowledge of definitions, mathematical notions, terms, statements of rules, theorems provided by the program; ability to prove them;
- ability of accurate expressing mathematical opinion in oral and written form by using necessary mathematical symbols;
- confident practical mathematical skills provided by the program, ability to use it while mathematical problems solving.

I. FUNDAMENTAL MATHEMATICAL NOTIONS AND FACTS.

ARITHMETIC, ALGEBRA AND PRECALCULUS

1. Natural numbers and null. Reading and writing of natural numbers. Natural numbers congruence. Addition, subtraction, multiplication and division of natural numbers.
2. Natural numbers divisibility. Divisors and multiples of a natural number. Paired and unpaired numbers. Divisibility by 2,3,5,9,10. Division with a remainder. Prime and composite numbers. Natural number factorization. Greatest common divisor, lowest common multiple.
3. Common fractions. Common fractions comparison. Proper and improper fraction. An integral and fractional part of a number. Fractional property. Reduction of fractions. Arithmetic average of a few numbers. Basic mathematical fractional problems.
4. Percentage. Rules of percentage calculations. Simple and compound percentage formulas.
5. Numerical inequality properties.
6. Arithmetic and geometric progression. N-th term formula and a sum of progression first terms. Formula of sum to geometric series terms with a denominator.
7. Power with natural and rational exponent. Arithmetical root.
8. Monomial and polynomial. Operations on it. Abridged multiplication formulas.
9. Polynomial with one variable. Root of polynomial (by quadratic trinomial).

10. An equation. Solution of the equation, roots of the equation. An equivalent equation. A graph of a binary equation.
11. Inequality. Solution of inequalities. Equivalent inequalities.
12. Systems of equations and inequalities. Systems solution. Roots of systems. Equivalent systems of equations.
13. Formula of quadric equation roots.
14. Quadratic trinomial factorization into linear multipliers.
15. Function. Representation of function. Domain, range of function. Reciprocal function.
16. Graph of function. Function increase and falling; periodicity, parity, oddity of function.
17. Function properties, its graph.
18. Function properties, its graph.
19. Function properties, its graph.
20. Definition and properties of functions. Functions graphs.
21. Reduction formulas.
22. Dependence between trigonometric functions of the same argument.
23. Sine and cosine of 2 arguments sum and difference (formulas).
24. Trigonometric functions of double argument.
25. Productization.
26. Solution of an equation.
27. Definition and properties of functions: power and exponential function.
28. A logarithm and its properties. Main logarithmic identity.
29. The logarithm of product, power and quotient.
30. Definition and properties of the logarithmic function.
31. Definition of a derivative, its mechanical and geometric content.
32. Derivative of sum, product and quotient of 2 functions.
33. Derivative of blending function.
34. Derivatives of trigonometric functions.
35. Derivatives of power, exponential and logarithmic functions.

36. Tangent equation to a graph of function.
37. Sufficient condition of function increase (decrease) on the interval. Extremum of function. Necessary condition for extremum. The largest and the smallest value of function on the interval.
38. Primitive and definite integral. Table of primitive elementary functions. Rules of antiderivation. Newton-Leibniz formula.
39. Permutations (without repetitions), rule of permutation. Arrangements (without repetitions), number of arrangements. Combinations (without repetitions).
40. Newton binomial. Probability of an accidental event. Elementary cases of probabilities count.
41. Static characteristics of data sequences.

GEOMETRY

1. Line, ray, segment, kinked curve; length of line. Equality and similarity of geometric figures. Relation of similar figures areas.
2. Angle, amount of angle. Vertical and contiguous angles. Parallel lines.
3. Triangle. Median, bisectrix, altitude of triangle, their characteristics. Types of triangles.
4. Features of triangles equality.
5. Characteristics of isosceles triangle.
6. Sum of angles of triangle. Sum of interior angles of convex polygon.
7. Features of parallel lines in the plane.
8. Circumference and circle. Center, diameter, radius, chords, secants of circumference.
9. Circumference circumscribed about triangle.
10. Circumference inscribed in triangle.
11. Tangent to circumference and its characteristic.
12. Characteristics of the points equidistant from the endpoints of segment.
13. Quadrangle: parallelogram, rectangle, rhomb, square, trapezium; their main characteristics.

14. Features of parallelogram.
15. Pythagorean theorem, conclusions of Pythagorean theorem.
16. Relation between sides and angles of right-angled triangle.
17. Distance formula between two points of plane. Circumference equation.
18. Examples of geometric figures transformations, types of symmetry.
19. Vectors. Operations with vectors.
20. Features of similarity of triangles.
21. Central and inscribed angles; their characteristics. Measurement of angle inscribed in circumference.
22. Polygon. Vertices, sides, diagonals of polygon.
23. Dependence between segments in circumference. Arc of circumference. Sector, segment.
24. Length of circumference and length of arc of circumference. Radian measure of angle. Area of circle and area of sector.
25. Formulas of areas of geometric figures: triangle, rectangle, parallelogram, square, rhomb, trapezium.
26. Plane. Parallel planes and intersecting planes.
27. Parallelism of straight line and plane.
28. Feature of planes parallelism.
29. Parallelism of straight lines and planes.
30. Features of parallelism of straight line and plane.
31. Perpendicularity of straight lines and planes.
32. Theorem on perpendicularity of straight line and plane.
33. Perpendicularity of two planes.
34. Angle of straight line to plane. Perpendicular to plane.
35. Dihedral angles. Linear angle of dihedral angle. Perpendicularity of two planes.
36. Polygons. Vertices, edges, faces, diagonals of polygon. Straight line and inclined line of prism. Pyramid. Regular prism and regular pyramid. Parallelepipeds, their types.

37. Bodies of revolution: cylinder, cone, sphere, ball. Center, diameter, radius of sphere and ball. Plane tangent to sphere.
38. Formulas of surface area and volumes of prism, pyramid, cylinder, cone.
39. Formulas of surface area of sphere, volume of ball.

II. MAIN FORMULAS AND THEOREMS

Algebra

1. Formula of n-th term of arithmetical and geometrical progression.
2. Formula of sum of first terms of arithmetical and geometrical progression.
3. Function, its characteristics and graph.
4. Function, its characteristics and graph.
5. Function, its characteristics and graph.
6. Function, its characteristics and graph.
7. Formula of roots of quadratic equation.
8. Expansion of quadratic trinomial to linear multipliers.
9. Formulas of abridged multiplication.
10. Properties of numerical inequalities.
11. Formulas of logarithms.
12. Functions, their definitions, characteristics and graphs.
13. Functions, their definitions, characteristics and graphs.
14. Formulas for solution of main trigonometrical equations.
15. Formulas of dependence between trigonometrical functions of the same argument, trigonometrical functions of double argument.
16. Reduction formulas.

Introduction to Analysis

1. Derivative of sum, product and quotient of two functions.
2. Derivatives of power, trigonometrical, exponential and logarithmic function.
3. Equation of tangent to function graph.
4. Main formulas of integration.
5. Applying the definite integral to calculate area of plane figure. Newton-Leibniz formula.

6. Calculation of combinations without repetition.
7. Newton's binomial formula.
8. Main theorems on probability of events. Bernoulli's formula.

Geometry

1. Characteristics of triangle. Equality and similarity, sides of triangle, angles of triangle, area of triangle – formulas of calculation, right-angled triangle, isosceles triangle, altitude of triangle.
2. Correlation between sides and angles of right-angled triangle.
3. Quadrangle. Main characteristics, formulas and theorems.
4. Circumference circumscribed about triangle and inscribed in triangle. Tangent to circumference and its characteristics. Theorem on angle inscribed in circumference.
5. Rectangular coordinate system. Formula of distance between two points of the plane. Equation of circumference.
6. Features of parallelism: straight line and plane, planes.
7. Theorem on perpendicularity of straight line and plane. Perpendicularity of two planes.
8. Formulas of areas of surface and volume of prism, pyramid, cylinder, cone.
9. Formulas of area of sphere and volume of ball.

III. MAIN KNOWLEDGE AND SKILLS

The entrant must be able to:

- carry out arithmetic operations with natural numbers, decimal fractions, simple fractions;
- carry out identical transformations of main algebraic expressions (polynomials, rational expressions which include power, exponential, logarithmic and trigonometric functions);
- construct and read graphs of linear, quadratic, power, exponential, logarithmic and trigonometric functions;
- solve equations and inequalities of first and second power as well as equations and inequalities reduced to them. Simplest equations and

inequalities which have power, exponential, logarithmic and trigonometric functions;

- solve problems on addition of equations and systems of equations;
- have practical skills of basic mathematical analysis, use derivative while investigating functions, use integral for calculation of areas of plane figures;
- carry out operations with vectors and use them when solving practical problems and exercises;
- apply characteristics of geometric figures when solving problems.

IV. TYPICAL PROBLEMS IN THE INTERVIEW

CALCULATIONS AND IDENTICAL TRANSFORMATIONS

1. Simplify the expression:

$$a) \left(\frac{b^{\frac{5}{6}} b^{\frac{1}{3}}}{b^{\frac{7}{18}}} \cdot \frac{b^{\frac{2}{7}}}{b^{\frac{5}{7}}} \right)^9; \quad б) \frac{\cos(\alpha + \beta) + \sin \alpha \sin \beta}{\sin(\alpha - \beta) - \sin \alpha \sin \beta}$$

2. Calculate:

$$a) \left(\frac{8^{\frac{1}{2}} \cdot 9^{\frac{4}{3}}}{27^{-\frac{1}{9}} \cdot 4^{\frac{1}{4}}} \right)^{-1}; \quad б) \frac{\log_9 27 + \log_9 3}{2 \log_2 6 - \log_2 9}; \quad в) \log_7 x = \log_7 2,5 + 4 \log_7 2 - \log_7 10;$$

$$г) \cos\left(\frac{\pi}{6} + \alpha\right), \text{ якщо } \sin \alpha = -\frac{\sqrt{3}}{3}; \quad \pi < \alpha < \frac{3\pi}{2}.$$

3. Prove identity:

$$a) \frac{\sin \alpha}{1 + \cos \alpha} + \frac{1 + \cos \alpha}{\sin \alpha} = -\frac{2}{\sin \alpha};$$

$$б) \sin(\alpha - \pi) + \operatorname{tg}(\alpha - \pi) + \cos\left(\frac{3\pi}{2} + \alpha\right) = \operatorname{tg} \alpha.$$

EQUATIONS AND SYSTEMS OF EQUATIONS

$$4. \sqrt{x + 78} - x = 6; \quad 5. 5^{x+2} + 5^x = 130; \quad 6. 3^{2x+1} + 8 \cdot 3^x - 3 = 0;$$

$$7. \lg(2x - 1) + \lg(x - 9) = 2; \quad 8. \log_7^2 x - \log_7 x^2 - 3 = 0;$$

$$9. \sin^2 x + 2 \sin x \cos x - 3 \cos^2 x = 0; \quad 10. \sin 3x + \sin x = 0.$$

$$11. \begin{cases} \frac{x}{y} - \frac{y}{x} = \frac{16}{15}; \\ x - y = 2 \end{cases}; \quad 12. \begin{cases} \sqrt[3]{x} - \sqrt[3]{y} = 2; \\ x - y = 56 \end{cases}; \quad 13. \begin{cases} 3^{2x-y} = 81 \\ \lg xy = 1 + \lg 3 \end{cases}$$

INEQUALITIES AND SYSTEMS OF INEQUALITIES

$$14. \sqrt{x^2 - x} > 2\sqrt{3}; \quad 15. \left(\frac{1}{3}\right)^x < \frac{1}{81}; \quad 16. 2^{x^2 - 8x + 19} > 16;$$

$$17. \log_3(x-3) > 0; \quad 18. \log_{\frac{1}{5}}(3x-5) > \log_{\frac{1}{5}}(x+1);$$

$$19. \sin x < \frac{1}{2}; \quad 20. \frac{x^2 - x - 2}{x^2 - 25} \leq 0; \quad 21. \begin{cases} x^2 - 7x - 18 \geq 0 \\ 2x + 3 > 0 \end{cases}$$

INTRODUCTION TO ANALYSIS

22. Find function domain:

$$a) f(x) = \frac{\sqrt{5 - 4x - x^2}}{x + 2};$$

$$b) f(x) = \sqrt{\frac{(x+4)(3-x)}{\lg(x^2+1)}}.$$

23. Find derivative value in a point:

$$a) f(x) = \frac{1}{x^4} - \frac{2}{x^3}, x_0 = 1;$$

$$b) f(x) = \cos 6x, x_0 = \frac{\pi}{12}.$$

24. Find monotony intervals and extremum points of function:

$$a) f(x) = \frac{x^2 + 7x}{x - 9};$$

$$b) f(x) = 2x^4 - 2x^3 - x^2 + 2.$$

25. Find the largest and the smallest function value $f(x) = \frac{x^3}{3} - 3x^2 + 5x - 7$

on the interval $[-1; 3]$.

26. Find equation of tangent line to graph of function $f(x) = 4x - \frac{x^3}{3}$

at the point $x_0 = 3$.

27. Find the original of function $f(x) = 5x^4 + 3x^2 - 4$ at the point $A(-1; 12)$.

28. Evaluate the integral:

$$a) \int_{-1}^{\frac{1}{2}} (x^2 - 4x + 5) dx;$$

$$b) \int_0^{\frac{\pi}{2}} \left(2 \cos 2x + \frac{1}{3} \sin \frac{x}{3} \right) dx.$$

29. Evaluate the area of the figure bounded by given lines:

$$y = x^2 - 4x + 5, y = 5 - x.$$

SOLUTION OF TRIANGLES

30. Solve a right-angled triangle by a hypotenuse and an acute angle.

Given: $c = 12,6, A = 64^{\circ}30'$. Find B, b, a .

31. Solve a right-angled triangle by two legs.

Given: $a = 56,2; b = 29,8$. Find: A, B, c .

32. Solve a triangle by the two sides and the angle between them.

Given: $a = 76,8; b = 56,4; C = 62^{\circ}20'$. Find: B, A, c .

33. Solve a triangle by two angles and a side.

Given: $a = 6,15; A = 36^{\circ}8'; C = 62^{\circ}20'$. Find: B, b, c .

PLANE VECTORS AND COORDINATES

34. Determine the inner product of vectors $3\vec{a} + 2\vec{b}$ та $\vec{a} - \vec{b}$, if, $\vec{a}(0;1), \vec{b}(2;1)$.

35. Check if the points $A(-4;-4), B(-3;4), C(4;5), D(10;-2)$ are vertices of trapezium.

36. Determine the angle A of triangle ABC , if the coordinates of the vertices are known: $A(1;3), B(4;6), C(3;1)$.

GEOMETRIC OBJECTS AND SURFACES

37. The right-angled parallelepiped have sides 7 and 24 dm, its height is 0,8 meters. Determine the area of diagonal section.

38. Cylinder inscribed into right triangular prism have sides 13, 14 and 15 cm, the height is 20 cm. Evaluate the cross-section of cylinder.

39. Lateral edge of the right four-cornered pyramid is 5 cm and its height is 3 cm. Find the volume of the pyramid.

40. The area base of cone is square $9\pi \text{ cm}^2$ and the area of its total lateral surface is square $24\pi \text{ cm}^2$. Find the volume of the cone.

41. The surface area of a sphere is square $225\pi \text{ cm}^2$. Determine its volume.

V. EVALUATION CRITERIA

The entrance exam in mathematics is conducted in the form of a written test.

The given time for examination is 3 academic hours. Subject commission had developed variants of tasks; each of them contains 32 tests, distributed into three levels of complexity. All tasks from the first part are «closed-ended». There are four possible variants for each of the tasks (A,B,C,D), only the one is correct. These tasks based on algorithmic nature and their solution requires the entrant's ability to recognize basic mathematical objects and carry out tasks according known algorithms in ordinary situations. The task from this part is considered to be done correctly, if there is only one letter in the answer table, which marked as correct answer. In addition to that, the entrant should not give any reasons to explain his choice. He should fill in only the second line of the table in the answer sheet:

Number of the task	1	2	3	4	5	6	7	8	9	10
Correct answer	A	A	A	A	A	A	A	A	A	A
	B	B	B	B	B	B	B	B	B	B
	C	C	C	C	C	C	C	C	C	C
	D	D	D	D	D	D	D	D	D	D
Teacher's point										

Number of the task	11	12	13	14	15	16	17	18	19	20
Correct answer	A	A	A	A	A	A	A	A	A	A
	B	B	B	B	B	B	B	B	B	B
	C	C	C	C	C	C	C	C	C	C
	D	D	D	D	D	D	D	D	D	D
Teacher's point										

The first line in the table is the number of the task. In the second line an entrant, writes down the letter that corresponds to the answer of the certain problem. In the third line a teacher, who checks the work indicates the correctness of the executed task: puts "+" if it is correct, and "-" if the answer is wrong or missing. In this part, every correct answer is estimated at five points. Therefore, the maximum number of points that an entrant can receive while solving the task from the first part of the exam is one hundred (100) points.

The second part contains ten tasks of sufficient complexity and evaluates as follows: for every right answer, an entrant will get 8 points. While solving the tasks of this level they should be accompanied by necessary arguments (explanation) and a picture (if necessary). The answer writes down separately. In this part, the maximum number of points that an entrant can get for correctly done tasks is one eighty (80) points.

The third part contains two tasks of a high level of difficulty, each of which is evaluated in 10 points, provided a thorough explanation of the solution and the correct answer. The maximum number of points that an entrant can receive for correctly completed tasks of this part twenty (20) points.

During evaluation the tasks that had been done wrong in the second and third part should be done next: if an entrant found the right way how to solve it, but he had made arithmetic (technical) error, as the result he should not loss more than 50% of points for the task. If the solution is incomplete, but contains at least one step that confirms the knowledge of necessary entrant formulas, theorems and possession of skills, as a result he should not loss more than 75% of points for the task. But if there is no solving, and the answer recorded correctly, it can be evaluated in five points in the second and zero points in third part.

The scale of test evaluation into five and two hundred point systems.

Five-point grading scale	Two hundred-point grading scale
1	1-50
2	51-99
3	100-149
4	150-189
5	190-200

Criteria for evaluating educational achievements of students

Number of points	Evaluation criteria
1-50	Entrant: <ul style="list-style-type: none"> - doesn't know conceptual apparatus - can not explain how to solve tasks even with assistance - doesn't know about the actual content of the material
51-99	Entrant: <ul style="list-style-type: none"> - has shown weak skills in conceptual and terminological apparatus - can explain the ways how to solve problems only with the help of teacher

100-149	<p>Entrant:</p> <ul style="list-style-type: none"> - answers some questions - independently but incompletely reproduces the educational material - generally uses correct mathematical terms - solves basic and typical mathematical problems
150-189	<p>Entrant:</p> <ul style="list-style-type: none"> - demonstrates knowledge of factual material, but makes some inaccuracies - solves proposed tasks, but can make some arithmetical inaccuracies - corrects errors
190-200	<p>Entrant:</p> <ul style="list-style-type: none"> - has excellent skills in conceptual and terminological apparatus - have shown the ability to solve the proposed problems without any mistakes - argues and proves all the statements

Each member of the subject commission evaluates the level of entrant's knowledge according to the evaluation criteria. The general evaluation level of knowledge is derived as a result of the discussion between the members of commission concerning personal marks. The commission makes a decision: «recommended to be enrolled» or «not recommended to be enrolled».